11-4 Irrational Square Roots

Objective: To simplify radicals and to find decimal approximations of irrational square roots.

Vocabulary

Irrational numbers Real numbers that can't be expressed in the form $\frac{a}{b}$, where a and b are integers. Their exact values can't be expressed as either terminating or repeating decimals.

Property

Property of Completeness Every decimal represents a real number, and every real number can be represented by a decimal.

Example 1   Simplify: **a.** $\sqrt{256}$ **b.** $\sqrt{50}$ **c.** $2\sqrt{80}$ **d.** $\sqrt{704}$

Solution   **a.** $\sqrt{256} = \sqrt{4 \cdot 64}$ Factor within the radical sign.

$\quad\quad\quad\quad = \sqrt{4} \cdot \sqrt{64}$ Use the product property of square roots.

$\quad\quad\quad\quad = 2 \cdot 8$ Simplify.

$\quad\quad\quad\quad = 16$

b. $\sqrt{50} = \sqrt{25 \cdot 2}$

$\quad\quad\quad = \sqrt{25} \cdot \sqrt{2}$

$\quad\quad\quad = 5\sqrt{2}$

c. $2\sqrt{80} = 2\sqrt{16 \cdot 5}$

$\quad\quad\quad\quad = 2 \cdot 4\sqrt{5}$

$\quad\quad\quad\quad = 8\sqrt{5}$

d. $\sqrt{704} = \sqrt{64 \cdot 11}$

$\quad\quad\quad\quad = 8\sqrt{11}$

Simplify.

1. $\sqrt{27}$ **2.** $\sqrt{20}$ **3.** $\sqrt{72}$ **4.** $\sqrt{32}$ **5.** $\sqrt{48}$

6. $\sqrt{45}$ **7.** $\sqrt{196}$ **8.** $\sqrt{80}$ **9.** $2\sqrt{63}$ **10.** $4\sqrt{98}$

11. $7\sqrt{28}$ **12.** $4\sqrt{40}$ **13.** $\sqrt{441}$ **14.** $\sqrt{289}$ **15.** $3\sqrt{50}$

16. $12\sqrt{50}$ **17.** $\sqrt{729}$ **18.** $\sqrt{432}$ **19.** $8\sqrt{75}$ **20.** $2\sqrt{90}$

21. $\sqrt{147}$ **22.** $\sqrt{288}$ **23.** $\sqrt{4225}$ **24.** $5\sqrt{800}$ **25.** $5\sqrt{1025}$

11-5 *Square Roots of Variable Expressions*

Objective: To find square roots of variable expressions and to use them to solve equations and problems.

Property

Property of Square Roots of Equal Numbers For any real numbers r and s:
$r^2 = s^2$ if and only if $r = s$ or $r = -s$.

CAUTION When you are finding the principal square root of a variable expression, you must be careful to use absolute value signs when needed to ensure that your answer is positive. For example, $\sqrt{x^2} = |x|$, not x.

Example 1 Simplify: **a.** $\sqrt{144x^2}$ **b.** $\sqrt{25n^8}$ **c.** $\sqrt{12a^3}$

Solution **a.** $\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$

$\qquad = 12|x|$

b. $\sqrt{25n^8} = \sqrt{25} \cdot \sqrt{n^8}$

$\qquad = \sqrt{25} \cdot \sqrt{(n^4)^2}$

$\qquad = 5n^4$ (n^4 is always nonnegative)

c. $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a}$

$\qquad = \sqrt{4} \cdot \sqrt{a^2} \cdot \sqrt{3} \cdot \sqrt{a}$

$\qquad = 2|a|\sqrt{3a}$

Simplify.

1. $\sqrt{81x^2}$

2. $\sqrt{121x^2}$

3. $\sqrt{20x^2}$

4. $\sqrt{45x^4}$

5. $-\sqrt{25x^2}$

6. $-\sqrt{16c^4}$

7. $-\sqrt{64d^8}$

8. $-\sqrt{98n^6}$

9. $\sqrt{225y^4}$

10. $\sqrt{400a^6b^4}$

11. $\sqrt{81m^{12}}$

12. $\sqrt{441n^6}$

13. $\pm\sqrt{75x^2y^3}$

14. $\pm\sqrt{60x^6y^4}$

15. $-\sqrt{121x^2y^2}$

16. $-\sqrt{900a^4b^6}$

17. $\pm\sqrt{\dfrac{81x^8}{100}}$

18. $\pm\sqrt{\dfrac{121}{225x^{10}}}$

19. $\sqrt{\dfrac{x^4y^8}{9z^2}}$

20. $\sqrt{\dfrac{32m^3n^2}{2mn^2}}$

21. $\sqrt{\dfrac{16x^{18}}{3600y^{20}}}$

22. $\sqrt{\dfrac{256x^{40}}{25}}$

23. $\sqrt{2.25x^4}$

24. $-\sqrt{2.56k^2}$